



Mark Scheme (Unused)

January 2022

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$\frac{2}{\sqrt{9-2x}} = \frac{2}{3\sqrt{\left(1-\frac{2}{9}x\right)}}$ or $\frac{2}{\sqrt{9-2x}} = 2(9-2x)^{-\frac{1}{2}} = 2 \times \frac{1}{3} \left(1-\frac{2}{9}x\right)^{-\frac{1}{2}}$	Obtains $\sqrt{9-2x} = 3\sqrt{\left(1-\dots\right)}$	B1
	$\left(1-\frac{2}{9}x\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{2}{9}x\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{2}{9}x\right)^2 + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{2}{9}x\right)^3 + \dots$ M1: Attempts the binomial expansion of $(1+kx)^n$ to get the third and/or fourth term with an acceptable structure. The correct binomial coefficient must be combined with the correct power of x and the correct power of 2. A1: Correct simplified or unsimplified expansion (NB simplified is $= 1 + \frac{1}{9}x + \frac{1}{54}x^2 + \frac{5}{1458}x^3 + \dots$)		M1 A1
	$\frac{2}{\sqrt{9-2x}} = \frac{2}{3} + \frac{2}{27}x + \frac{1}{81}x^2 + \frac{5}{2187}x^3 + \dots$	2 correct simplified terms	A1
		All correct	A1
			(5)
(b)	$x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\Rightarrow \sqrt{7} \approx 2 \div \frac{1652}{2187} \text{ " or } 2 \times \frac{2187}{1652} \text{ "}$ Substitutes $x = 1$ and divides into 2 or equivalent		M1
	$= 2.6477$	Correct approximation	A1
			(2)
	Alternative for (b):		
	$x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7} \Rightarrow \sqrt{7} \approx \frac{7}{2} \times \frac{1652}{2187} \text{ "}$ Substitutes $x = 1$ and multiplies by $\frac{7}{2}$		M1
	$= 2.6438$	Correct approximation	A1
			Total 7

Question Number	Scheme	Notes	Marks
2(a)	$\frac{x}{y} = t$	Cao	B1
			(1)
(b)	$y = \frac{\left(\frac{x}{y}\right)^3}{2\left(\frac{x}{y}\right)+1}$ or $x = \frac{\left(\frac{x}{y}\right)^4}{2\left(\frac{x}{y}\right)+1}$	Uses the y coordinate to obtain y in terms of x and y or uses the x coordinate to obtain x in terms of y and x	M1
	$y = \frac{x^3}{2xy^2 + y^3} \Rightarrow y(2xy^2 + y^3) = x^3$ or $x = \frac{x^4}{2xy^3 + y^4} \Rightarrow x(2xy^3 + y^4) = x^4$	Uses correct algebra to eliminate the fractions	M1
	$x^3 - 2xy^3 - y^4 = 0^*$	Cso	A1*
			(3)
			Total 4

Question Number	Scheme	Notes	Marks
3(a)	$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$ $\Rightarrow \underline{6y \frac{dy}{dx} - 22x + 11x \frac{dy}{dx} + 11y} = 20 \frac{dy}{dx} - 36$ M1: $y^2 \rightarrow Ay \frac{dy}{dx}$ M1: $11xy \rightarrow px \frac{dy}{dx} + qy$ A1: All correct		M1M1A1
	$(6y + 11x - 20) \frac{dy}{dx} = 22x - 11y - 36 \Rightarrow \frac{dy}{dx} = \dots$ Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject		M1
	$\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20}$	Correct expression or correct equivalent	A1
			(5)
(b)	$x = 4 \Rightarrow 3y^2 - 176 + 44y = 20y - 144 + 28$	Substitutes $x = 4$ into C to obtain a 3TQ in y	M1
	$3y^2 + 24y - 60 = 0 \Rightarrow y = \dots$	Solves for y	M1
	$y = -10 \text{ (}, 2)$	Correct value	A1
	$(4, -10) \rightarrow \frac{dy}{dx} = \frac{88 + 110 - 36}{-60 + 44 - 20}$	Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{9}{2}$	Correct value	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	$\frac{4-4x}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$	Correct form for the partial fractions	B1
	$4-4x = A(x-2)^2 + Bx(x-2) + Cx$ $\Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	Uses a correct strategy to find at least one of their constants	M1
	$\frac{4-4x}{x(x-2)^2} \equiv \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$	2 correct constants	A1
		All correct	A1
			(4)
(b)	$\int \left(\frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2} \right) dx = \ln x - \ln(x-2) + \frac{2}{x-2} (+c)$		M1
	M1 for $\int \frac{\alpha}{x} dx = \beta \ln x$ or $\int \frac{\alpha}{x-2} dx = \beta \ln(x-2)$		M1
	M1 for $\int \frac{\alpha}{(x-2)^2} dx = \frac{\beta}{x-2}$		A1
	A1: All correct		(3)
(c)	$\left[\ln x - \ln(x-2) + \frac{2}{x-2} \right]_3^5 = \left(\ln 5 - \ln 3 + \frac{2}{3} \right) - (\ln 3 - \ln 1 + 2)$ $= \ln \frac{5}{9} - \frac{4}{3}$		M1
	M1: Correct use of limits and reaches the required form using log rules		A1
	A1: Correct answer		(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots$ or $\mu = \dots$	Full method for finding λ or μ	M1
	$\lambda = 2, \mu = -1$	Both correct values	A1
	$-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda = 2 \rightarrow (4 + 4)\mathbf{i} + (4 - 6)\mathbf{j} + (-5 + 12)\mathbf{k}$ or $\mu = -1 \rightarrow (13 - 5)\mathbf{i} + (-1 - 1)\mathbf{j} + (4 + 3)\mathbf{k}$	Uses their λ or μ to find A .	M1
	$8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos \theta = \pm \frac{11}{7\sqrt{35}}$	Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$)	A1
	$\theta = 74.6^\circ$	Awrt 74.6	A1
			(3)
(c)	$ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$	Finds the magnitude of the direction of l_1	M1
	$35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18, -17, 37)$ or $(-2, 13, -23)$	One correct point (ignore labels)	A1
	$P(18, -17, 37)$ and $Q(-2, 13, -23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			Total 6

Question Number	Scheme	Notes	Marks
7(a)	$\frac{dV}{dt} = 300 - kV \Rightarrow \int \frac{dV}{300 - kV} = \int dt$	Correct separation of variables	B1
	$\int \frac{dV}{300 - kV} = -\frac{1}{k} \ln(300 - kV)$	$\int \frac{dV}{300 - kV} = \alpha \ln(300 - kV)$	M1
	$-\frac{1}{k} \ln(300 - kV) = t + c$	Correct equation including a constant of integration	A1
	$-\frac{1}{k} \ln(300 - kV) = t + c \Rightarrow \ln(300 - kV) = -kt + d$ $\Rightarrow 300 - kV = e^{-kt+d}$ Correct processing to remove the “ln”		M1
	$kV = 300 - e^{-kt+d} \Rightarrow V = \frac{300}{k} - Be^{-kt}$ $V = \frac{300}{k} + Ae^{-kt} *$	Correct proof	A1*
			(5)
(b)	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$V = \frac{300}{k} - \frac{300}{k} e^{-kt} \Rightarrow \frac{dV}{dt} = 300e^{-kt}$	$\frac{dV}{dt} = \alpha e^{-kt}$	M1
	$300e^{-10k} = 200 \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ and correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
			(4)
(b) Way 2	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$\frac{dV}{dt} = 200, t = 10 \Rightarrow 200 = 300 - kV$ $\Rightarrow kV = 100$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ to find a value for kV	M1
	$V = \frac{300}{k} + Ae^{-kt} \Rightarrow kV = 300 - 300e^{-10k}$ $\Rightarrow 100 = 300 - 300e^{-kt} \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Substitutes for kV, kA and $t = 10$ and uses correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
(c)	$6000 = \frac{3000}{\ln 1.5} - \frac{3000}{\ln 1.5} e^{-\frac{t}{10} \ln 1.5}$ $\Rightarrow e^{-\frac{t}{10} \ln 1.5} = 1 - 2 \ln 1.5$ $\Rightarrow -\frac{t}{10} \ln 1.5 = \ln(1 - 2 \ln 1.5)$	Correct strategy using $V = 6000$ to reach $at = \dots$	M1
	$t = 41$	Correct value	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
8	Assume that there exist positive real numbers x and y such $\frac{9x}{y} + \frac{y}{x} < 6$	Starts the proof by contradicting the given statement	B1
	$\frac{9x}{y} + \frac{y}{x} < 6 \Rightarrow 9x^2 + y^2 < 6xy$ as x and y are both positive	Multiplies through by xy	M1
	$\Rightarrow 9x^2 + y^2 - 6xy < 0$ $\Rightarrow (3x - y)^2 < 0$	Reaches a correct contradictory statement	A1
	As x and y are positive real numbers, this is a contradiction and so $\frac{9x}{y} + \frac{y}{x} < 6$ must be incorrect and so $\frac{9x}{y} + \frac{y}{x} \dots 6^*$	Makes a suitable conclusion	A1*
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
9(a)	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta$ $= \pi \int (3 \sin \theta - \sin 2\theta)^2 (-5 \sin \theta) d\theta$	Applies $V = \pi \int y^2 \frac{dx}{d\theta} d\theta$ with or without the π	M1
	$= \pi \int (3 \sin \theta - 2 \sin \theta \cos \theta)^2 (-5 \sin \theta) d\theta$	Applies $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
		Fully correct integral in terms of $\sin \theta$ and $\cos \theta$ only (π not needed)	A1
	$= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta *$	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u = \cos \theta \Rightarrow V = 5\pi \int \sin^3 \theta (3 - 2u)^2 \frac{du}{-\sin \theta}$	Applies the substitution correctly	M1
	$\theta = 0 \Rightarrow u = 1, \theta = \pi \Rightarrow u = -1$	Attempts to change θ limits to u limits	M1
	$V = -5\pi \int \sin^2 \theta (3 - 2u)^2 du = -5\pi \int (1 - u^2)(3 - 2u)^2 du$ Correct integral in terms of u only		A1
	$(1 - u^2)(3 - 2u)^2 = (1 - u^2)(9 - 12u + 4u^2)$ $= 9 - 12u - 5u^2 + 12u^3 - 4u^4$	Attempt to expand	M1
		Correct expansion	A1
	$V = 5\pi \int_{-1}^1 (9 - 12u - 5u^2 + 12u^3 - 4u^4) du$ $= 5\pi \left[9u - 6u^2 - \frac{5u^3}{3} + 3u^4 - \frac{4u^5}{5} \right]_{-1}^1 = \dots$ Integrates and applies their u limits		M1
	$= \frac{196}{3} \pi$	Cao	A1
			(7)
			Total 11